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Optimum Mixing of Core and Bypass Streams in High-Bypass Civil Turbofan

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In a mixed-stream turbofan, except for fan pressure ratio, the mission matched optimum selection of other basic cycle variables like bypass ratio, overall pressure ratio, throttle ratio, and turbine entry temperature depends upon engine-airframe interactions over the prescribed mission and the available technology level. The fan pressure ratio is decided by the mixing conditions that are internal to the engine. This paper, by utilizing a multidisciplinary cycle optimization software, performs a number of numerical experiments to determine a criteria for mixing of bypass and core streams, which results in an optimum fan pressure ratio. The use of optimum fan pressure ratio will aid the other basic cycle parameters in enhancing performance gains caused by mixing, thereby improving the engine-airframe-mission compatibility. The penalty of non- or suboptimal mixing, and savings in mission fuel consumption for a mixed-stream turbofan with respect to a separate exhaust turbofan, both at optimum fan pressure ratio are also quantified.

Nomenclature

$\dot{A}$	=	area, m^2
D	=	diameter, m
$ER1 \ldots ER4$	=	cycle balancing errors
H	=	altitude, km
L	=	length, m
M	=	Mach number
P	=	total pressure, Pa
p	=	static pressure, Pa
S_{LND}	=	landing ground run, m
S_{TO}	=	takeoff ground run, m
S_W	=	aircraft wing area, m^2
T	=	total temperature, K
t	=	time, s
W	=	engine mass flow rate, kg/s
W_{EMP}	=	aircraft empty weight, kg
$W_{\mathrm{ENG,DP}}$	=	engine design mass flow, kg/s
$W_{F,\mathrm{msn}}$	=	mission fuel consumed, kg
W_{TO}	=	aircraft takeoff gross weight, kg
γ	=	ratio of specific heats
σ	=	pressure loss coefficient

Subscripts

CL	=	climb
c	=	cold/bypass flow
DP	=	design point
h	=	hot/core flow
max	=	maximum value of a variable
mix	=	mixed-out condition

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I. Introduction

MIXED-STREAM turbofan engines in which the core and bypass streams are mixed before expansion is now increasingly being viewed as a candidate for civil aircraft as well, besides extensive military application. In comparison to the separate exhaust turbofan, the major advantage of a mixed-stream concept is an improvement of about 2–3% in specific thrust and thrust specific fuel consumption (SFC).[1,2] Although marginal, it is significant over the entire operation life of the aircraft. A theoretical and experimental description of the mixing process and the basis and magnitude of resulting performance gains with respect to a separate exhaust turbofan are contained in Refs. 1–3.

There are a total of six primary cycle variables that define a twin-spool, mixed-stream turbofan concept for civil use. They are bypass ratio (BPR), fan pressure ratio (FPR), low-pressure compressor pressure ratio (PRLC), overall pressure ratio (OPR), maximum turbine entry temperature ($\mathrm{TET_{max}}$), and throttle ratio ($\mathrm{TR} = \mathrm{TET_{max}}/\mathrm{TET_{DP}}$). Assigning a numerical value to each of them creates an engine cycle option. The optimum cycle is the one that minimizes mission fuel consumed ($W_{F,\mathrm{msn}}$), while satisfying the thrust demand of all of the mission segments and various size and performance constraints to ensure its functional feasibility and aerothermal-mechanical design compatibility.

Except for FPR, the optimum selection of remaining primary cycle variables is driven by integrated engine-airframe-constraints interactions over the prescribed mission application. As described in the following section on "mixer considerations," choice of FPR depends upon the conditions at which the core and bypass streams are mixed, which are purely internal to the engine and independent of mission application. Therefore, if mixing of core and bypass streams is done under conditions that result in an optimum FPR it will further aid in improving the fuel efficiency and mission adaptability of those primary cycle variables, the optimum for which are strongly mission dependent.

A. Mixer Considerations

In a mixed-stream turbofan engine static pressures of the core and bypass streams at the mixer inlet (p_h and p_c) must be made equal

$(p_c = p_h)$ at all of the flight points, prior to mixing. If the engine state at a given station is known (that is, total temperature T, total pressure P, and mass flow rate W), then specifying a Mach number M enables the computation of the static pressure of the flow and the area required to pass it, using the standard gas-dynamic equations.

Thus, when all of the cycle variables are prescribed at the design point, Mach number at the mixer inlet plane for the core $(M_{h,\mathrm{DP}})$ and bypass streams $(M_{c,\mathrm{DP}})$ needs to be specified to compute their respective static pressures and also the mixer inlet core (A_h) and bypass (A_c) flow areas. FPR is iterated until $p_c = p_h$ is achieved, typically within $\pm 0.001\%$. The A_h and A_c are then assumed fixed and are used to compute off-design Mach numbers, and hence the mixer inlet static pressures of the core and bypass streams, to determine off-design FPR. Summarizing, the thermodynamic performance of an engine cycle in presence of mixer is defined by $M_{h,\mathrm{DP}}$ and $M_{c,\mathrm{DP}}$, which not only define the design FPR but also its off-design variations.

Having selected $M_{h,\mathrm{DP}}$ and $M_{c,\mathrm{DP}}$, an important dependent variable is the design point ratio of total pressures of bypass $(P_{c,\mathrm{DP}})$ and core stream $(P_{h,\mathrm{DP}})$ at mixer inlet, that is, $(P_c/P_h)_{\mathrm{DP}}$. In fact, out of $M_{h,\mathrm{DP}}$, $M_{c,\mathrm{DP}}$ and $(P_c/P_h)_{\mathrm{DP}}$, any two can be selected, and the third one satisfying the static pressures equality is obtained from Eq. (1). It can be seen that [Eq. (1)] at a prescribed $M_{h,\mathrm{DP}}$, $M_{c,\mathrm{DP}}$ increases with increase in $(P_c/P_h)_{\mathrm{DP}}$, and irrespective of the numerical value of $M_{h,\mathrm{DP}}$, $M_{c,\mathrm{DP}}$ and $M_{h,\mathrm{DP}}$ are nearly equal at $(P_c/P_h)_{\mathrm{DP}} = 1.0$.

$$M_c = \left(\frac{2}{\gamma_c - 1}\left\{\left[\frac{P_c}{P_h}\left(1 + \frac{\gamma_h - 1}{2}M_h^2\right)^{\gamma_h/(\gamma_h - 1)}\right]^{(\gamma_c - 1)/\gamma_c} - 1\right\}\right)^{\frac{1}{2}} \tag{1}$$

B. Scope of the Present Work

Reference 2 is one of the early works which states that for optimum mixer performance $P_{c,\mathrm{DP}}$ should not be lower than $P_{h,\mathrm{DP}}$ and should not exceed it by 10–20%. A similar criteria is identified in Ref. 4, which indicates that $(P_c/P_h)_{\mathrm{DP}}$ should be near unity, that is, for near equal values of $M_{h,\mathrm{DP}}$ and $M_{c,\mathrm{DP}}$. The $(P_c/P_h)_{\mathrm{DP}}$ of unity can arise at various values of $M_{c,\mathrm{DP}}$ (or $M_{h,\mathrm{DP}}$). But, none of these references present the sensitivity of cycle fuel efficiency to $M_{c,\mathrm{DP}}$ at the optimum $(P_c/P_h)_{\mathrm{DP}}$. While Ref. 2 is based on experimental data, Ref. 4 is a numerical study based on heuristic approach.

This paper uses $M_{h,\mathrm{DP}}$ and $M_{c,\mathrm{DP}}$ as primary cycle variables to define the mixer performance and utilizes an alternative multidisciplinary cycle optimization approach to reascertain the optimality criteria of Refs. 2 and 4. Besides, it also attempts to identify an optimum $M_{c,\mathrm{DP}}$ to complement the optimum $(P_c/P_h)_{\mathrm{DP}}$. The penalty of deviating from this optimal criteria has also been evaluated. The paper concludes with a case study to illustrate the potential benefits in the form of saving in $W_{F,\mathrm{msn}}$ when a mixed-stream turbofan is used instead of a separate exhaust turbofan, both at optimum FPR.

II. Cycle Optimization

A nonlinear constrained optimization problem is solved to locate the value of an n-dimensional vector of design variables, that is, $X = (x_1, x_2, \ldots, x_n)$ that minimizes aircraft takeoff gross weight (W_{TO}) because a smaller aircraft costs less to build and operate. The "optimization with surface fits"[5,6] is used, where system response instead of being called directly from design simulator,[6] and is made available to the optimizer (complex method of Box)[7] as surface fits $\{y_i = f(x_1, x_2, \ldots, x_n)$, y_i being ith response variable$\}$. For any combination of design variables, surface fits act as fast analysis modules, from which response is obtained quickly, unlike the time intensive design simulator. Thus a large number of optimization problems are easily solved in a reasonable time. To generate surface fits, a selective number of design combinations are first identified within a certain design space.[8] The response is then computed for each of them using the design simulator, and finally regression analysis[9] is performed on the resulting data.

The success of present work largely depends on getting a good surface fit approximation of system response. For each response variable three checks were performed to ascertain the goodness of its surface fit approximation:

1) The multiple correlation coefficient is greater than 0.99.

2) The difference between the actual response (from design simulator) and fitted response at design combinations used for the development of surface fits is within $\pm 2\%$ of actual response and also within $\pm 2\%$ of the range of actual response (that is, the difference between the maximum and minimum values).

3) The difference between the actual and fitted response at a large number of nonregression design combinations (that is, the ones not used in the development of surface fits), well dispersed over the design space, is within $\pm 2\%$ of actual response.

As the final step, the optimum was accepted only if the difference between the actual and fitted response at the optimum is within $\pm 2\%$. However, this difference was observed to be less than $\pm 1\%$ most of the times. These aforesaid checks induce sufficient confidence in the validity of optimum solutions, which were obtained using the surface fit approximations of actual response and presented later in this paper.

III. Design Simulator

The design simulator is central to the cycle optimization software because it supplies system response by evaluating the mission performance of an engine cycle. It is an integrated computer simulation of aircraft equations of motion, airframe design characteristics, and engine steady-state thermodynamic performance to perform drag/thrust matching and fuel consumption at every mission flight point. The outcome of design simulator are mission matched values of engine design point mass flow $(W_{\mathrm{ENG,DP}})$ and W_{TO} such that the thrust demand of the most constraining segment is met, and aircraft consumes all of the fuel except reserves while flying the mission. The most constraining segment is the one that demands maximum thrust and therefore determines $W_{\mathrm{ENG,DP}}$. This $W_{\mathrm{ENG,DP}}$ then together with prescribed engine power settings is used to compute the performance of remaining segments.

The design simulator for military application, described in Ref. 6, was suitably modified for a civil aircraft study. It was validated with actual flight simulator data over a domestic sector for a modern 160 seater aircraft, which uses mixed-stream turbofans. The fuel consumed for each mission segment, total mission fuel, and mission time as computed by design simulator are in good agreement (within $\pm 2\%$) with actual flight simulator results. $W_{\mathrm{ENG,DP}}$ also works out to be close to actual engine. It induces sufficient confidence in design simulator accuracy.

A. Design Mission

Aircraft carries a payload of 12,000 kg. It takes off, climbs to $H = 10$ km/$M = 0.78$ in max power, and begins to cruise at this H/M for 4800 km, using 90% of max available thrust. The cruise thrust demand gradually decreases as the fuel is consumed and aircraft becomes lighter. The reserve fuel is accounted by cruise at $H = 3$ km/$M = 0.78$ for 240 km and loiter at optimum M at $H = 0.50$ km for 30 min. The engine power for these segments is adjusted based on thrust demanded/thrust available. Finally, the aircraft descends and lands.

B. Design Vector

To limit the problem size, only those variables having high sensitivity on system response are included in the design vector. A reduced problem size also aids in improving the quality of surface fit that approximate the true system response. To estimate engine cycle's interactions with airframe, airframe design variables also need to be included in design vector.

1. *Engine Design Variables*

Among the primary cycle variables, only BPR, OPR, $\mathrm{TET}_{\max}$, $M_{c,\mathrm{DP}}$, and $M_{h,\mathrm{DP}}$ are included in design vector. FPR is a dependent variable. Although a primary cycle variable, PRLC has a poor sensitivity on optimum W_{TO} and hence was assigned a value of 2.20 that is typical of current design trends. In a civil turbofan, only a limited flat rating up to a hot day condition of ISA (international standard atmosphere) + 20–30 K is required. Thus TR was also kept fixed at a moderate value of 1.04.

The remaining variables like component's efficiency, total pressure loss, customer and cooling bleeds, etc., are referred as secondary variables. It is advantageous to maximize components' efficiency and minimize pressure loss and bleeds. Thus, their values are kept fixed as per the state of art, instead of being optimized.

2. Airframe Design Variables

The variation of lift, zero lift and induced drag coefficient with Mach number and drag rise caused by use of flaps and landing gear, which is typical of a civil aircraft, is assumed to be applicable to all the design combinations. The aircraft empty weight W_{EMP} was kept fixed at 42,175 kg (that of Airbus A-320) and $W_{F,msn}$ together with reserves defines the internal fuel capacity. This simplified airframe representation eliminates design variables such as aspect ratio, wing sweep, thickness, and taper ratio, etc., and leaves only design point thrust loading (TLDG) and wing loading (WLDG: W_{TO}/S_W, where S_W is the aircraft wing area) in the analysis. To compute mission matched $W_{ENG,DP}$, WLDG was chosen as the design variable.

C. Response Variables

The important response variables are takeoff ground run (S_{TO}), landing ground run (S_{LND}), time to climb (t_{CL}) to cruise altitude, $W_{F,msn}$, W_{TO}, $W_{ENG,DP}$, TLDG, and S_W. Whereas W_{TO} is used as the figure of merit, one or more of the remaining are imposed as constraints. Surface fits need to be developed for the figure of merit and all of the response variables used as constraints.

D. Engine Cycle Performance

In a twin-spool civil turbofan with mixed streams, air entering the engine splits into the bypass and core streams at the exit of fan. While the bypass stream passes through an annular bypass duct, the core stream goes through the booster or low-pressure (LP) compressor, core or high-pressure (HP) compressor, combustor, and the HP and LP turbines. The two streams are mixed downstream of LP turbine and expanded in a single convergent exhaust nozzle with fixed geometry. The single-stage fan, followed by a two- or three-stage LP compressor are on the same shaft, which are driven by a multistage LP turbine. The multistage HP compressor is driven by a single- or two-stage HP turbine. Fuel is injected in the combustor and burned to produce hot gas for driving turbines. There is no reheat.

Sea-level static in ISA is the engine design point. The preliminary design methods that operate without utilizing component maps to estimate design and off-design steady-state uninstalled thrust and SFC for a mixed-stream military turbofan are contained in Ref. 10. They can easily be translated to the case of a civil turbofan. For this purpose a total of four errors ($ER1 \ldots ER4$) are defined for off-design cycle balancing. The iteration parameters are continuously updated until each error is within $\pm 0.001\%$. Mixing is assumed to be complete and ideal, and PRHC denotes the pressure ratio of HP compressor.

ER1 : HP turbine flow error $\Longleftrightarrow$ TET

ER2 : HP spool work imbalance $\Longleftrightarrow$ PRHC

ER3 : Mixer inlet static pressures $\Longleftrightarrow$ FPR

ER4 : Nozzle entry mass flow error $\Longleftrightarrow$ BPR

The limiters $\mathrm{TET} \le \mathrm{TET}_{max}$ and $\mathrm{OPR} \le \mathrm{OPR}_{DP}$ are imposed to ensure a feasible solution. Because LP compressor and fan are on the same shaft, PRLC is computed by assuming enthalpy rise across LP compressor to be proportional to enthalpy rise across fan and then referencing it to the design point condition. An empirical correlation {Installation Penalty = f (flight M, BPR)}[11] is used to compute installed performance. The earlier mentioned design simulator validation is an indication of the correctness of installed thrust and SFC computations by the engine cycle software. Besides, it has also been validated independently with respect to the limited manufacturer's data for a reference engine.

IV. Problem Definition

Design mission was kept fixed and cruise at $H = 10\,\mathrm{km}/M = 0.78$ sizes $W_{ENG,DP}$.

Minimize W_{TO}, subject to:

(I) Box constraints, that is, design space, and

(II) Inequality constraints ($g_1 \ldots g_4$):

$$(g_1)\mathrm{TLDG} \le 0.30, \qquad (g_2)S_{TO} \le 1900\,\mathrm{m}$$

$$(g_3)S_{LND} \le 1600\,\mathrm{m}, \qquad (g_4)t_{CL}, \text{ sea level to cruise } H \le 1800\,\mathrm{s}$$

The design space for present work is

$$4.50 \le \mathrm{BPR} \le 7.00, \qquad 25.0 \le \mathrm{OPR} \le 40.0$$

$$1700\,\mathrm{K} \le \mathrm{TET}_{max} \le 1800\,\mathrm{K}, \qquad 0.20 \le M_{h,DP} \le 0.60$$

$$0.20 \le M_{c,DP} \le 0.60, \qquad 520\,\mathrm{kg/m^2} \le \mathrm{WLDG} \le 650\,\mathrm{kg/m^2}$$

V. Results and Discussion

This section describes the numerical studies based on ideal mixer assumption to determine a criteria for optimum mixing of core and bypass streams in a high-bypass civil turbofan. The resulting criteria has been translated in an equivalent $(P_c/P_h)_{DP}$ to conform with Refs. 2 and 4, which use $(P_c/P_h)_{DP}$ to define mixer performance. Besides, an optimum $M_{c,DP}$ has also been identified that coupled with $(P_c/P_h)_{DP}$ aids to further improve the cycle performance.

A. Locating Optimum $(P_c/P_h)_{DP}$

Cycle optimization was performed by holding $M_{c,DP}$ at a prescribed numerical value and allowing the remaining design variables to optimize. Beginning from its lower limit, $M_{c,DP}$ was varied in steps of 0.05. TET_{max} being a technology parameter, it always takes the upper limit of its design space if allowed to optimize. It was therefore kept fixed at a preassigned numerical value, representative of either the existing or an advanced projected technology level. The variation of optimum $(P_c/P_h)_{DP}$ that results in minimum W_{TO} with $M_{c,DP}$ at three levels of TET_{max} is shown in Fig. 1. Increasing TET_{max} from 1700 to 1800 K indicates a transition from existing to advanced technology. It can be seen that regardless of technology level, optimum $(P_c/P_h)_{DP}$ varies within 0.98 to 1.02 for a wide variation in $M_{c,DP}$ from 0.20 to 0.60.

In all of the preceding optimization studies, whereas BPR took a range of values within its design space, OPR always moved to its upper design space limit. The reason being that although increasing both BPR as well as OPR improves SFC, BPR does so at the cost of greater reduction in specific thrust, causing a greater increase in $W_{ENG,DP}$ (and hence in TLDG) to meet a prescribed thrust demand. Thus to maximize cycle fuel efficiency at relatively higher levels of

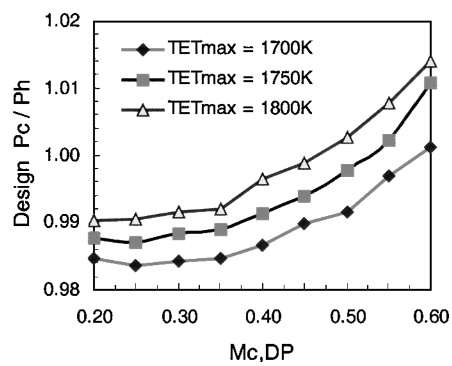

Fig. 1 Optimum $(P_c/P_h)_{DP}$.

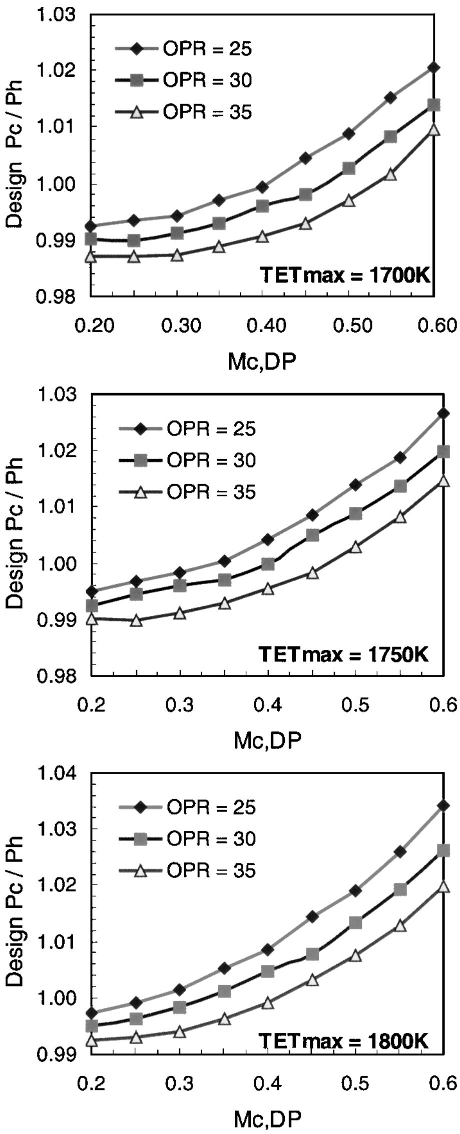

Fig. 2 OPR Sensitivity on optimum $(P_c/P_h)_{DP}$.

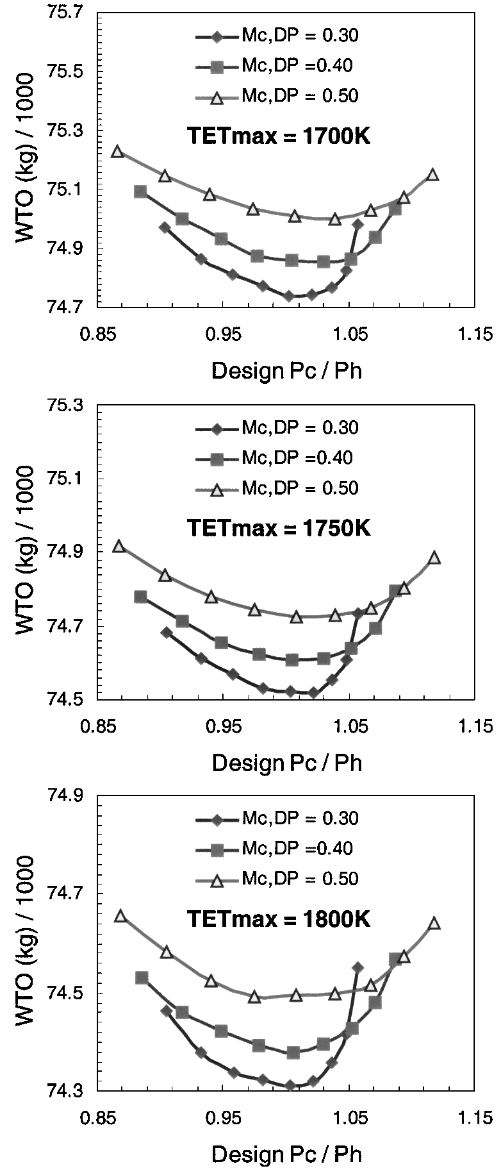

Fig. 3 $(P_c/P_h)_{DP}$ sensitivity on optimum W_{TO}.

specific thrust, OPR first moves to the highest possible value, and then BPR is determined by the level of TLDG constraint. The value that OPR takes depends upon the cycle TET_{max} and also on aircraft design cruise Mach number.

To determine the influence of variations in OPR on optimum $(P_c/P_h)_{DP}$, sensitivity studies were performed. For this purpose TET_{max} of 1700, 1750, and 1800 K were chosen. At each TET_{max}, sensitivity was investigated for OPR of 25, 30, and 35. At a given TET_{max} and for each OPR, $M_{c,DP}$ was varied in steps of 0.05 within its design space, and the remaining design variables were optimized. The resulting sensitivity trends shown in Fig. 2 indicate that irrespective of OPR, optimum $(P_c/P_h)_{DP}$ lies within 0.98–1.03.

Next, an attempt has been made to identify if variations in optimum $(P_c/P_h)_{DP}$ can be approximated to a single numerical value so as to uniquely define a $(P_c/P_h)_{DP}$ for optimum mixing of core and bypass streams. This value could typically be 1.0, being the average of the range of variation of optimum $(P_c/P_h)_{DP}$ from 0.98 to 1.02. Such an approximation is possible only if optimum W_{TO} has a poor sensitivity to $(P_c/P_h)_{DP}$ in the region of optimum variation so that shifting $(P_c/P_h)_{DP}$ from its actual optimum value to 1.0 does not reflect in any noticeable penalty in W_{TO}.

To generate the sensitivity of optimum W_{TO} to $(P_c/P_h)_{DP}$ at a given TET_{max}, $M_{c,DP}$ is assigned a fixed numerical value. The $M_{h,DP}$ is varied by ± 0.20 around this $M_{c,DP}$ in steps of 0.05. At the chosen TET_{max} and $M_{c,DP}$ the remaining design variables were optimized for each value of $M_{h,DP}$ that is obtained as described earlier, to get the minimum W_{TO}. If $M_{h,DP}$ falls outside its design space, surface fits were extrapolated. Using this procedure at each TET_{max} of 1700, 1750, and 1800 K, sensitivity of W_{TO} to $(P_c/P_h)_{DP}$ was obtained at $M_{c,DP}$ of 0.30 (low), 0.40 (medium), and 0.50 (high), and is contained in Fig. 3.

It is evident from Fig. 3 that variations in optimum W_{TO} are practically insignificant for $(P_c/P_h)_{DP}$ variation from 0.95 to 1.05, thereby indicating that W_{TO} has a poor sensitivity to $(P_c/P_h)_{DP}$ in the region of interest. Thus, $(P_c/P_h)_{DP}$ can be approximated to 1.00 as the unique condition for optimum mixing of core and bypass streams. The penalty of non- or suboptimal mixing at any TET_{max} and $M_{c,DP}$ can easily be quantified from Fig. 3.

B. Selecting Optimum $M_{c,DP}$

Having identified an optimum $(P_c/P_h)_{DP}$, it is necessary to complement it with an optimum $M_{c,DP}$ (or $M_{h,DP}$) to maximize the cycle performance. During the cycle optimization studies illustrated in Fig. 1, it was observed that at each TET_{max} the least of all of the optimum (minimum) W_{TO} as well as the least $W_{ENG,DP}$ corresponds to the lowest $M_{c,DP}$ of 0.20, as shown in Figs. 4 and 5. The $W_{ENG,DP}$ is consistent with empirical correlation stated in Eq. (2), which has been derived using a large database of currently manufactured turbofans engines with BPR of 2.0 and above,[12] thereby justifying the choice of TLDG constraint (≤ 0.30).

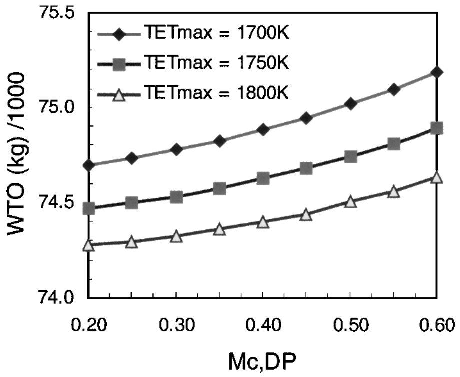

Fig. 4 Optimum W_{TO}.

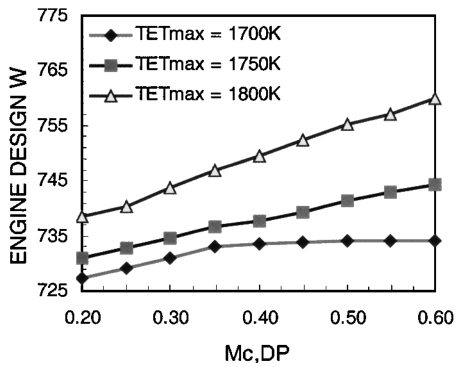

Fig. 5 $W_{\mathrm{ENG,DP}}$ vs $M_{c,\mathrm{DP}}$ at optimum.

$$W_{\mathrm{ENG,DP}}\ (\mathrm{lb/s}) = 0.032 \times \mathrm{Thrust}_{\text{Take-Off}}\ (\mathrm{lb}) \tag{2}$$

The trends of Figs. 4 and 5 indicate that for optimum mixing of core and bypass streams a low $M_{c,\mathrm{DP}}$ together with optimum $(P_c/P_h)_{\mathrm{DP}}$ of 1.0 is desirable. The loss in cycle fuel efficiency and thrust with increasing $M_{c,\mathrm{DP}}$ is explained by Eq. (3), which computes mixed-out total pressure P_{mix} after mixing of core and bypass streams. MFP denotes the mass flow parameter.

$$\frac{P_{\mathrm{mix}}}{P_h} = \frac{(1+\mathrm{BPR})}{1+A_c/A_h} \times \sqrt{\frac{T_{\mathrm{mix}}}{T_h}} \times \frac{\mathrm{MFP\ (mix)}}{\mathrm{MFP\ (core)}} \tag{3}$$

Consider a fixed cycle with ideal mixing and with design parameters of BPR = 5.0, PRLC = 2.20, OPR = 35.0, $\mathrm{TET}_{\max} = 1750$ K, TR = 1.04, and $(P_c/P_h)_{\mathrm{DP}} = 1.0$. During design point calculations in such a case, FPR always takes the same value irrespective of the level of $M_{c,\mathrm{DP}}$. Thus all of the cycle parameters upstream of mixer as well A_c/A_h, T_{mix}, and W_{mix} also take the same value and are independent of $M_{c,\mathrm{DP}} \cdot P_{\mathrm{mix}}$ now depends only upon the MFP ratio between the mixed and core streams. The design point variations of MFP ratio and the ratio of installed thrust and SFC (referenced to $M_{c,\mathrm{DP}} = 0.20$) with $M_{c,\mathrm{DP}}$ are contained in Fig. 6.

It shows that with increasing $M_{c,\mathrm{DP}}$, MFP ratio decreases, thereby reducing P_{mix} with increasing $M_{c,\mathrm{DP}}$. In practice, P_{mix} will further reduce with increasing $M_{c,\mathrm{DP}}$ as a result of frictional losses, which can be approximated by Eq. (4) (Ref. 13). Because P_{mix} governs the final nozzle pressure ratio, reduction in P_{mix} decreases thrust and increases SFC. A similar trend was observed at all other off-design mission flight points.

$$\sigma_{\mathrm{mixer}} = 0.05 \times M_{\mathrm{mix}}^2 \tag{4}$$

The foregoing discussion justifies the use of a low $M_{c,\mathrm{DP}}$, but it increases the mixer size. The fan entry being the maximum diameter section, mixing chamber diameter D_{mixer} must never exceed the fan frontal diameter D_{fan}. Thus choice of $M_{c,\mathrm{DP}}$ is a tradeoff between cycle thermodynamic performance and mixer size. The description of Table 1 enables us to understand this tradeoff, which contains the mixer geometry and weight at various $M_{c,\mathrm{DP}}$ for the optimum cycles at $\mathrm{TET}_{\max} = 1750$ K. Mixer is assumed to be an adiabatic cylindrical chamber, with cross-sectional area of $A_c + A_h$. Although a larger length L is required for complete mixing, mixer length-to-diameter L/D ratio is restricted to 0.75 as a result of space limitations in an actual engine hardware. The metal thickness and density are taken as 2.50 mm and 8200 kg/m^3, respectively, for mixer weight computation.

Table 1 Mixer size and weight for optimum at $\mathrm{TET}_{\max} = 1750$ K

Variables	I	II	III	IV	V	VI	VII	VIII	IX
$M_{c,\mathrm{DP}}$	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
A_c	2.52	2.05	1.74	1.52	1.36	1.24	1.15	1.08	1.02
A_h	0.63	0.54	0.48	0.43	0.40	0.37	0.35	0.33	0.32
D	2.00	1.82	1.68	1.58	1.50	1.43	1.38	1.34	1.31
L	1.50	1.36	1.26	1.18	1.12	1.07	1.04	1.01	0.98
Weight	98.0	81.0	69.0	61.0	55.0	50.0	47.0	44.0	42.0

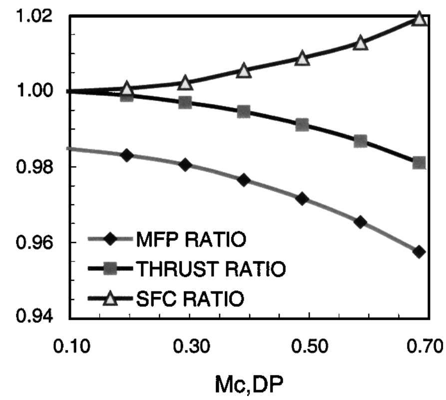

Fig. 6 $M_{c,\mathrm{DP}}$ influence on mixing.

Because all of the optimum engine cycles are in the class of 110 kN (25,000 lb) thrust, D_{fan} is of the order of 1.60 m, computed from statistical correlations of Ref. 12. In terms of cycle design mass flow to fan frontal area ratio, it is equivalent to about 185 kg/m^2, which is consistent with high-bypass turbofan designs. Thus all those $M_{c,\mathrm{DP}}$ for which D_{mixer} exceeds D_{fan} are ruled out. $M_{c,\mathrm{DP}} = 0.35$ is the lowest value that meets this constraint, but there is not much allowance between the fan and mixer diameters. Therefore $M_{c,\mathrm{DP}}$ of 0.40 is deemed suitable to complement the optimum $(P_c/P_h)_{\mathrm{DP}}$ of 1.0. The similar trends were also observed for optimum cycles at $\mathrm{TET}_{\max}$ of 1700 and 1800 K.

As cycle performance deteriorates with increasing $M_{c,\mathrm{DP}}$ and there is not much reduction in mixer diameter and weight beyond $M_{c,\mathrm{DP}}$ of 0.40, it is desirable to use $M_{c,\mathrm{DP}} = 0.40$. Moving from $M_{c,\mathrm{DP}}$ of 0.20 to 0.40 reduces mixer weight by 40 kg, which will marginally compensate the increase in $W_{F,\mathrm{msn}}$ as a result of loss in cycle fuel efficiency. If the engine is designed for cycle mass flow to frontal area ratio higher than 185 kg/m^2, that is, for a lower D_{fan}, then $M_{c,\mathrm{DP}}$ might need to be increased. As an example, if this ratio increases to 225 kg/m^2 as the limiting case then $D_{\mathrm{fan}} \approx 1.45$ m for the optimum cycles at $\mathrm{TET}_{\max}$ of 1750 K. Thus, $M_{c,\mathrm{DP}}$ of at least 0.45 is required.

C. Mixed vs Separate Exhaust

Given in Table 2 is a relative comparison of the optimum cycles (all at optimum FPR) in the mixed and separate exhaust modes at $\mathrm{TET}_{\max}$ of 1750 and 1800 K. The W_{EMP} (=42,175 kg), design space and problem formulation were kept the same as described earlier to derive these optimum. Mixing is assumed to be complete and ideal at $M_{c,\mathrm{DP}}$ of 0.40.

The mixed-stream optimum cycle has a lower FPR and a higher BPR. The reason being that for a given cycle restriction of equal mixer inlet static pressures at $(P_c/P_h)_{\mathrm{DP}}$ of 1.0 always results in a lower optimum FPR for the mixed-stream turbofan. It therefore

Table 2 Optimum cycles; mixed and separate exhaust turbofan

	Unmixed		Mixed	
Variables	I	II	III	IV
TET_{max}	1,750	1,800	1,750	1,800
BPR	5.33	5.88	5.51	6.13
OPR	40.0	40.0	40.0	40.0
FPR	1.787	1.791	1.686	1.694
PRLC	2.20	2.20	2.20	2.20
TR	1.04	1.04	1.04	1.04
$(P_c/P_h)_{DP}$	1.19	1.20	1.00	1.00
$W_{ENG,DP}$	742	747	739	749
WLDG	579	581	578	581
W_{TO}	75,520	75,316	74,627	74,401
$W_{F,msn}$	21,175	20,970	20,282	20,057

produces a higher specific thrust at lower SFC at all flight conditions. Because of higher specific thrust, mixed-stream turbofan is able to operate at a relatively higher BPR to meet the same thrust demand, thereby further improving the cycle fuel efficiency. At the same level of components' efficiency, pressure loss, and bleeds, mixing of core and bypass streams results in a saving of 1.18% in W_{TO} and 4.20% in $W_{F,msn}$ at TET_{max} of 1750 K, which slightly increase to 1.21 and 4.35% at TET_{max} of 1800 K.

VI. Conclusions

An improvement in the cycle performance of a separate exhaust turbofan can be achieved only by improving the components' efficiency and reducing the pressure loss and bleeds, which is a difficult and costly proposition. Mixing of core and bypass streams before expansion is a simpler solution to this problem. To maximize the potential benefits of mixing, optimum mixing conditions need to be identified in terms of $(P_c/P_h)_{DP}$ and either of $M_{c,DP}$ and $M_{h,DP}$. This paper describes a large number of numerical cycle optimization experiments based on ideal mixer assumption and integrated engine-airframe-mission interactions to arrive at the optimum mixing criteria in terms of $(P_c/P_h)_{DP}$ and $M_{c,DP}$.

The $(P_c/P_h)_{DP}$ that minimizes W_{TO} (and hence $W_{F,msn}$) over a prescribed typical civil transport mission is the optimum. It was observed that optimum $(P_c/P_h)_{DP}$ lies within 0.98–1.03 for a wide variation in primary engine cycle variables. It can be approximated to 1.0 to uniquely define this parameter for optimum mixing of core and bypass streams because optimum W_{TO} has a poor sensitivity to $(P_c/P_h)_{DP}$ in the range $0.95 \leq (P_c/P_h)_{DP} \leq 1.05$.

The optimum choice of $M_{c,DP}$, instead of being a mathematically defined optimum like $(P_c/P_h)_{DP}$, is a tradeoff between the cycle thermodynamic performance and mixer size. Although a low $M_{c,DP}$ is desirable, it can result in mixing chamber diameter being more than fan frontal diameter. For the existing design trends where the ratio of cycle design mass flow to fan frontal area is of the order of 185 kg/m^2, $M_{c,DP} = 0.40$ has been identified as the best compromise. $M_{c,DP}$ might need to be increased if fan is designed for a lower diameter, that is, for a higher ratio of cycle design mass flow to frontal area.

In a given cycle optimum FPR is always lower for a mixed-stream turbofan, in comparison to a separate exhaust turbofan. Besides, mixed-stream turbofan takes a higher BPR at the optimum designs. The net effect is an improvement in its fuel efficiency over all of the flight conditions.

Identification of a unique $(P_c/P_h)_{DP}$ and a suitable $M_{c,DP}$ for optimum mixing also simplifies the cycle optimization analysis as these variables can now be assigned fixed numerical values. It will reduce the problem dimension and hence the number of parametric design combinations at which response needs to be obtained for generating surface fit approximations of desired response variables. Alternatively, design parameters from other disciplines like airframe or mission can be included.

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